STRING FRAGMENTATION MODEL WITH SPINNING QUARKS [1]

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Abstract

The quark spin degree of freedom is introduced in the string fragmentation model, using Pauli spinors and matrices. The hadron mass-shell constraints, which were omitted in a preliminary model, are now satisfied. The algorithm for a recursive Monte-Carlo generation of a polarized quark jet is described.

1 Introduction

Quark spin plays a dynamical role in jet formation, as confirmed by the Collins effect. The Collins asymmetry can be used as a *quark polarimeter* for transversity. Similarly, *jet handedness* provides a polarimeter for quark helicity. To optimize these polarimeters, a theoretical model is needed as a guide. Since helicity and transversity are non-commuting observables, a model describing both effects must start with *quantum amplitudes* rather than probabilities. In this direction a toy model was proposed in [1]. This model, which uses Pauli spinors, not only reproduces the transverse spin effects of the classical string + $^3P_0$ mechanism [2,3], but yields jet handedness in addition. However, it does not take into account the hadron mass-shell constraints. This approximation allows a full decoupling of longitudinal and transverse momenta and makes analytical calculations possible, but is too crude for realistic Monte-Carlo simulations of jets.

In this paper we propose a model with mass-shell constraints. It combines the spin factors of the toy model and the kinematical dependance of the string fragmentation model [2]. In Section 2 we review the two main models of quark jets without spin : the *ordinary recursive model* and the *string fragmentation model.* In Section 3 we review the toy model of [1]. In Section 4 we write the quantum amplitudes underlying the string fragmentation model and include spin matrix factors in them. In Section 5 we give the Monte-Carlo algorithm for a recursive processing of the model.

2 Spinless fragmentation models

Fig.1a depicts an event of e^+e^- annihilation or $W^\pm$ decay into quark q_A + antiquark $\bar{q}_B$, followed by the hadronisation process

$$q_A + \bar{q}_B \rightarrow h_1 + h_2 ... + h_N \, . \tag{1}$$

Hadrons at the right and left sides form the *quark* and *antiquark* jets. Here we will restrict ourselves to processes without hard gluon and without initial or final baryon.

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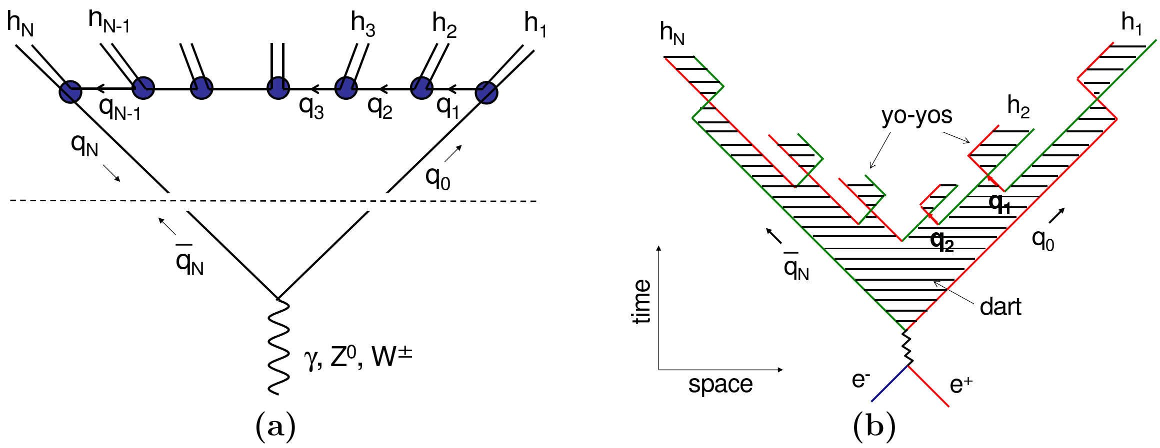


Figure 1: **(a)** $\mathrm{e}^+\mathrm{e}^-$ annihilation or $W^\pm$ decay in quark-antiquark $\to$ hadrons. **(b)** String fragmentation.

The simple recursive model. The most simple model of quark jets for Monte-Carlo simulations is the *recursive model* [4, 5]. Looking from right to left at the upper part of Fig.1a, the process (1) can be decomposed in

$$q_0 \to h_1 + q_1 \;, \qquad q_1 \to h_2 + q_2 \;, \qquad \cdots\cdots \quad q_{N-1} \to h_N + q_B \,. \tag{2}$$

$q_0 \equiv q_A$ and $q_B \equiv q_N$ is the charge conjugate of $\bar{q}_B$ propagating "backward in time" with 4-momentum $q_B \equiv -\bar{q}_B$. The 4-momentum conservation $q_{n-1} = p_n + q_n$ holds at each step. p_n is the 4-momentum of n^{th}*-rank hadron.* q_n stands either for the species (u, d or s) of the n^{th}-rank quark or for its 4-momentum. In the simplest receipe, the sharing between p_n and q_n is made according to the *splitting probability distribution*,

$$d\zeta_n \, d^2\mathbf{q}_{n\mathrm{T}} \, f(\zeta_n, q_{n\mathrm{T}}) \;, \tag{3}$$

where $\mathbf{q}_\mathrm{T} = (q^x, q^y)$, $\zeta_n = q_n^+/q_{n-1}^+$ and $q^\pm \equiv q^0 \pm q^z$. The $+z$ and $-z$ directions are along $\mathbf{q}_\mathrm{A}$ and $\bar{\mathbf{q}}_\mathrm{B}$.

Including the quark *flavor* degree of freedom is relatively easy. The $q \to h + q'$ splitting function depends on the flavors and writes $f_{q',h,q}(\zeta, q'_\mathrm{T})$.

Notations. The symbol $\{q_n\}$, with curly brackets, represents the momentum *and* the flavor of the n^{th} quark altogether. See, *e.g.* Eq.(5). A four-momentum q is separated in transverse part $\mathbf{q}_\mathrm{T} = (q^x, q^y)$ and time-longitudinal part $q_\mathrm{L} = (q^0, q^z)$. The virtual mass square is $q^2 = q_\mathrm{L}{}^2 - \mathbf{q}_\mathrm{T}{}^2 = q^+ q^- - \mathbf{q}_\mathrm{T}{}^2$.

The polarization vector of a quark is decomposed as $\mathbf{S} = (S_\mathrm{L}, \mathbf{S}_\mathrm{T})$ where $S_\mathrm{L}/2 = \langle\text{helicity}\rangle$, $\mathbf{S}_\mathrm{T} = \langle\text{transversity}\rangle$. The density matrix is $\rho = (1 + \mathbf{S}.\vec{\sigma})/2$.

The string fragmentation model [2, 6, 7]. One may consider Fig.1a as a diagram of the dual resonance model. Hadronization is the cascade decay of a massive string (the *dart*) stretching between q_A and $\bar{q}_B$. The space-time picture is shown in Fig.1b. At the n^{th} string breaking point (starting from the right) a $q_n\bar{q}_n$ pair is created. $\bar{q}_n$ moves to

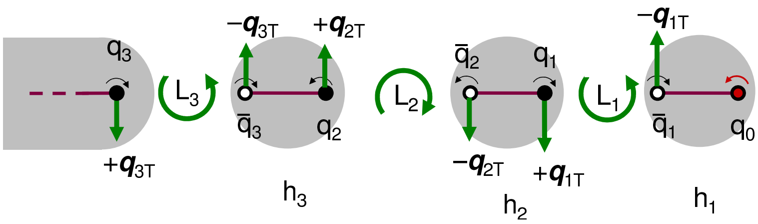


Figure 2: The string + ${}^3P_0$ mechanism for Collins effect.

the right, meets q_{n-1} which is moving to the left and both form the hadron h_n. If the null-plane coordinate $X^- = t - z$ is used as time variable, the hadrons are emitted in the ordering of (2) and the string model can be treated as a recursive one, with the *symmetric Lund splitting function* [2],

$$f_{q',h,q}(\zeta, \mathbf{q}'_\mathrm{T}, \mathbf{q}_\mathrm{T}) \propto Z^{a\{q\}}\ (1/Z-1)^{a\{q'\}}\ \exp\left(-b\,\frac{m_h^2+{\mathbf{p}_\mathrm{T}}^2}{Z}\right). \tag{4}$$

$Z = 1-\zeta$ and $a\{q\} \equiv a_q(\mathbf{q}_\mathrm{T}^2)$, which generally depends on the quark flavor q and transverse momentum $\mathbf{q}_\mathrm{T}$. Eq.(4) is used in the Monte-Carlo simulation code PYTHIA.

The string fragmentation model is invariant under
– (a) rotations about the z-axis,
– (b) Lorentz transformations along the z-axis
– (c) mirror reflection about any plane containing the z-axis (equivalent to parity),
– (d) *quark chain reversal* or "left-right symmetry", *i.e.,* interchanging q_A and $\bar{q}_B$.
It is not covariant *locally* (*i.e.,* step-by-step), but *globally* for the whole process of Fig.1.

3 Review of the toy model of Ref. [1]

The classical *string* + ${}^3P_0$ mechanism [2, 3]. We consider the simplest case where all the emitted partices are pseudoscalar mesons. Then $(q_n\bar{q}_{n-1})$ in h_n is a spin singlet. At a string breaking the $q_n\bar{q}_n$ pair is assumed to be created in the ${}^3P_0$ state with zero total momentum (corresponding to the vacuum quantum numbers). Fig.2 depicts the recursive decay of the dart when q_0 has a transverse, anti-clockwise polarization. $(q_0\bar{q}_1)$ is a spin-singlet, therefore $\bar{q}_1$ spins clockwise. $(q_1\bar{q}_1)$ is a spin-triplet, therefore q_1 also spins clockwise. Due to the ${}^3P_0$ configuration, the relative $q_1 - \bar{q}_1$ orbital momentum $\mathbf{L}_1$ is opposite to the spins, therefore anti-clockwise. It makes $\bar{q}_1$ move upward and q_1 move downward in the figure. The upward momentum of q_1 is taken by hadron h_1, resulting in a Collins effect, with $\mathbf{p}_{1\mathrm{T}}$ on the side of $\mathbf{S}_{0\mathrm{T}} \times \hat{\mathbf{z}}$.

Iterating this reasoning, q_2 and $\bar{q}_2$ are spinning anti-clokwise, $\mathbf{L}_2$ is clokwise, etc. One obtains Collins effects of alternate sides for h_2, h_3, etc. Of course, successive spins are not so rigidly coupled and the Collins effect decays along the quark chain. Nevertheless the model predicts a Collins effect for h_2 opposite to that of h_1 and reinforced by the fact that q_1 and $\bar{q}_2$ move on the same side. This is in agreement with experiment.

The string + ${}^3P_0$ mechanism also explains the polarization of inclusive hyperons [2].

The covariant quark-multiperipheral amplitude. The upper half of Fig.1a looks like a multiperipheral diagram [8], but with quark exchanges instead of meson exchanges. We treat q_A and $\bar{q}_B$ as on mass-shell quarks and assume that the probability of the whole process of Fig.1a factorizes in the probabilities of the upper and lower parts. The amplitude of (1) writes

$$\mathcal{M}\{q_A\bar{q}_B \to h_1 h_2 ... h_N\} = \\ \Gamma\{q_B, h_N, q_{N-1}\}\, \Delta\{q_{N-1}\} \cdots \Delta\{q_2\}\, \Gamma\{q_2, h_2, q_1\}\, \Delta\{q_1\}\, \Gamma\{q_1, h_1, q_A\}\,. \tag{5}$$

$\Delta\{q\} = D_q(q^2)\,(\mu_q + \gamma.q)$ is the quark propagator. μ_q is the quark mass. $D_q(q^2)$ is a fast decreasing function of $|q^2|$. $\Gamma\{q', h, q\} \equiv \Gamma_{q',h,q}(q', q)$ is the $q \to h + q'$ vertex function, which is a 4×4 matrix in the space of Dirac spinors. For the emission of a pseudoscalar meson, $\Gamma\{q', h, q\} = \gamma_5\, G_{q',h,q}(q'^2, q^2)$. The model is covariant *locally*, *i.e.*, at each vertex and propagator.

Another important approximation is to neglect interferences between several diagrams giving the same final state. Then the total hadronisation cross section writes

$$\sigma\{\bar{q}_B, q_A\} = \sum_N \sum_{h_1,...h_N} \int \frac{d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_N}{p_1^0 \cdots p_N^0}\, \delta^4(p_1 + p_2 ... + p_N - q_A - \bar{q}_B\} \\ |\bar{v}(\bar{q}_B, \mathbf{S}_\mathrm{B})\, \mathcal{M}\{q_A\bar{q}_B \to h_1 h_2 ... h_N\}\, u(q_A, \mathbf{S}_\mathrm{A})|^2\,. \tag{6}$$

The second summation bears on the hadron species. $u(q_A, \mathbf{S}_\mathrm{A})$ and $v(\bar{q}_B, \mathbf{S}_\mathrm{B})$ are the Dirac spinors of q_A and $\bar{q}_B$.

Reduction to Pauli spinors. We now describe the spin degree of freedom in the most economical way, with Pauli instead of Dirac spinors. We give up local covariance, but maintain the invariances (a), (b), (c) and (d) listed in section 2 about the string model. For this we replace [1]

- $u(q_0, \mathbf{S}_0)$ by the Pauli spinor $\chi(\mathbf{S}_0)$
- $\bar{v}(q_{\bar{q}_B}, \mathbf{S}_{\bar{q}_B})$ by $-\chi^\dagger(-\mathbf{S}_{\bar{q}_B})\,\sigma_z$
- γ_5 by σ_z
- $\mu_q + \gamma.q$ by $\mu_q + \sigma_z\,\sigma.\mathbf{q}_\mathrm{T}$.

Thus, the propagators has the non-covariant form

$$\Delta\{q\} = D_q({q_\mathrm{L}}^2, {\mathbf{q}_\mathrm{T}}^2)\; (\mu_q + \sigma_z\,\sigma.\mathbf{q}_\mathrm{T})\;. \tag{7}$$

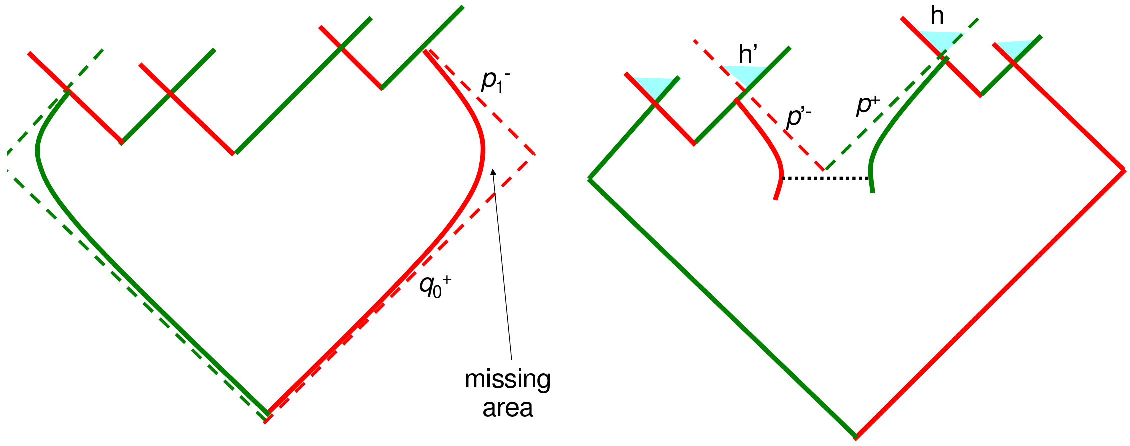
The toy model [1]. We consider only pseudo-scalar mesons, with the momentum-independent emission vertex σ_z, and take a factorized, flavor-independent quark propagator

$$\Delta\{q\} = D_L({q_\mathrm{L}}^2)\, \exp(-B{\mathbf{q}_\mathrm{T}}^2/2)\; (\mu + \sigma_z\,\sigma.\mathbf{q}_\mathrm{T})\,. \tag{8}$$

Furthermore we ignore the mass-shell constraint $m_n^2 = p_n^+ p_n^- - p_{n,\mathrm{T}}^2$. This crude approximation achieves the full decoupling of the longitudinal momenta from the transverse ones and from the quark spin. The joint $\mathbf{p}_T$ -distibutions of the n first mesons have simple expressions, for instance

$$I(\mathbf{p}_{1\mathrm{T}}, \mathbf{p}_{2\mathrm{T}}, \mathbf{p}_{3\mathrm{T}}) \propto \exp(-B{\mathbf{q}_\mathrm{T}}_1^2 - B{\mathbf{q}_\mathrm{T}}_2^2 - B{\mathbf{q}_\mathrm{T}}_3^2)\; \mathrm{Tr}\left\{\mathbf{M}_3\,\mathbf{M}_2\,\mathbf{M}_1\,\rho_0\,\mathbf{M}_1^\dagger\,\mathbf{M}_2^\dagger\,\mathbf{M}_3^\dagger\right\}, \tag{9}$$

where $\rho_0 = (1 + \mathbf{S}_0.\sigma)/2$ is the spin density matrix of q_0 and $\mathbf{M}_n = (\mu + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}n})\,\sigma_z$. For complex μ one obtains a Collins effect for each meson, the analyzing power of which depends only on the meson rank. See Ref. [1] for more properties of the model.



(a) **(b)**

Figure 3: **(a)** Trajectories of massive q_A and $\bar{q}_B$. **(b)** Tunneling trajectories of q_n and $\bar{q}_n$.

4 The semi-quantized string model

Let us first consider spinless quarks and mesons. Following the sum-over-histories approach of Feynman, to the classical string history of Fig.1b we associate the amplitude

$$\begin{gathered}\mathcal{M}(q_A\bar{q}_B \to h_1 h_2 ... h_N) = \exp[\,(-i\kappa_C + 2i\kappa)\,\mathcal{A}\,] \\ (q_A^+ p_1^-)^{\alpha\{q_A\}}\;(-p_1^+ p_2^- - i0)^{\alpha\{q_1\}}\;\cdots\;(-p_{N-1}^+ p_N^- - i0)^{\alpha\{q_{N-1}\}}\;(p_N^+ \bar{q}_B^{\,-})^{\alpha\{q_B\}} \\ g\{q_B, h_N, q_{N-1}\}\cdots g\{q_2, h_2, q_1\}\,g\{q_1, h_1, q_0\}\,. \end{gathered} \tag{10}$$

• $\mathcal{A}$ is the space-time area swept by the dart. $\kappa_C = \kappa - i\mathcal{P}/2$ is the *complex* string tension of the dart [9], accounting for its unstability (in analogy with the complex mass $m - i\Gamma/2$ of an unstable particle). We will use $b \equiv \mathcal{P}/(2\kappa^2)$.

The exponent of the first line contains the pure *string action* of the dart (proportional to $-\kappa_C$) and "missing propagation phases" (proportional to 2κ) of the final hadrons, taking into account their different emission points [10].

• The first and last power-law factors of the 2$^{\rm nd}$ line takes into account the *quark actions* of q_A and $\bar{q}_B$, which in the case of non-zero mass follow the pieces of hyperbolas in Fig.3a. We have

$$\alpha\{q_A\} = (b - i/\kappa)\,\mu_A^2\,/2 \quad (\text{idem for } \bar{q}_B). \tag{11}$$

These factors also take into account the "missing string area" [7] between the hyperbolas and the brokenline trajectories that would be followed by massless quarks.

• The intermediate power-law factors of the 2$^{\rm nd}$ line take into account the actions of the quarks and antiquarks created in pairs at string ruptures (Fig.3b). They simulate a multi-Regge behavior at large rapidity gaps, $\alpha\{q\}$ being the quark Regge trajectory. One may take the analytic continuation of (11), replacing q_A^+ by $-p_n^+$, p_1^- by p_{n+1}^- :

$$\alpha\{q_n\} = (\mu_n^2 + \mathbf{q}_{n\mathrm{T}}^2)\,(b - i/\kappa)/2\,. \tag{12}$$

For real μ_n the modulus square of the $n^{\rm th}$ factor is

$$(p_n^+ p_{n+1}^-)^{b(\mu_n^2 + \mathbf{q}_{n\mathrm{T}}^2)}\,\exp[-\pi(\mu_n^2 + \mathbf{q}_{n\mathrm{T}}^2)/\kappa]\,, \tag{13}$$

which exhibits the characteristic exponential factor of Schwinger tunneling [2]. This tunneling is represented by a dotted line in Fig.3b. There is however a limitation to Eq.(12). The tunneling length is $2E_{q\mathrm{T}}/\kappa = 2(\mu^2 + \mathbf{q}_\mathrm{T}^2)^{1/2}/\kappa$. It must be smaller than the string length, which is of the order of $\mathcal{P}^{-1/2}$. In fact the production of large E_T quarks should not be described by the string model, but by perturbative QCD. Besides, at large rapidity gap (large $p_n^+ p_{n+1}^-$), the first factor of (13) would too much favor heavy quarks.

A possible sensible choice is to use Eq.(12) with $b = 0$.

- The last line contains vertex functions

$$g\{q', h, q\} \equiv g_{q',h,q}({\mathbf{q}_\mathrm{T}'}^2, \mathbf{q}_\mathrm{T}'.\mathbf{q}_\mathrm{T}, {\mathbf{q}_\mathrm{T}}^2) \tag{14}$$

which depend on flavours and *transverse* momenta, but not on longitudinal ones. Quark chain reversal imposes g to be symmetric under the interchange $\{q; \mathbf{q}_T\} \leftrightarrow \{q'; \mathbf{q}'_T\}$.

Taking the modulus square of (10) for the fully differential cross section of (1) is equivalent to the symmetric Lund model.

Inclusion of quark spin. Spin is simply included by inserting the 2×2 matrices of the toy model. Fig.4. indicates where such matrices operate. Restricting ourselves to pseudoscalar meson production, we have to multiply the expression (10) by the chain of 2×2 matrices

$$\sigma_z\,(\mu_{N-1} + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}N-1})\,\sigma_z \cdots$$
$$(\mu_2 + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}2})\,\sigma_z\,(\mu_1 + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}1})\,\sigma_z\,. \tag{15}$$

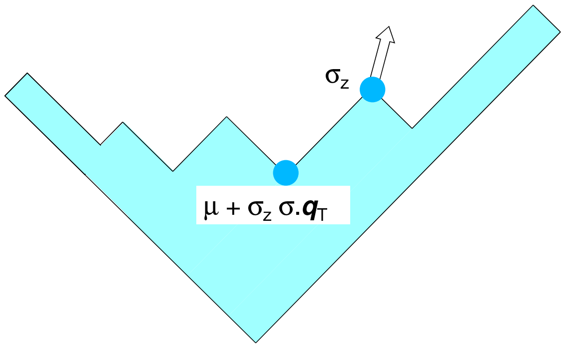


Figure 4: Spin matrices to be inserted in the string amplitude.

To sum up, the cross section of (1) with polarized q_A and $\bar{q}_B$ is given by

$$\sigma\{\bar{q}_B, q_A\} = \sum_N \sum_{h_1,\ldots h_N} \int \mathrm{d}^4q_1 \cdots \mathrm{d}^4q_{N-1}\; 2\delta(p_1^2 - m_1^2)\cdots 2\delta(p_N^2 - m_N^2)$$
$$\left|\chi^\dagger(-\mathbf{S}_\mathrm{B})\,\sigma_z\,\mathcal{M}\,\chi(\mathbf{S}_\mathrm{A})\right|^2, \tag{16}$$

$\mathcal{M}$ being given by (10) times (15). Unlike the toy model, the present string fragmentation model takes into account the mass-shell conditions properly.

5 Recursive Monte-Carlo Algorithm

- The string amplitude (10) times (15) can be put in a multiperipheral form. The *splitting amplitude*, defined as the product of the n^{th} vertex and the n^{th} propagator, is given by

$$T_n \equiv T\{q_n, h_n, q_{n-1}\} \equiv \Delta\{q_n\}\,\Gamma\{q_n, h_n, q_{n-1}\} =$$
$$\exp\left(\frac{i-b}{2}\,q_{n-1}^+ p_n^-\right)(q_{n-1}^+ p_n^-)^{\alpha\{q_{n-1}\}}\left(\frac{-p_n^+ - i0}{q_n^+}\right)^{\alpha\{q_n\}} g\{q', h, q\}\,(\mu_n + \sigma_z\sigma.\mathbf{q}_{n\mathrm{T}})\,\sigma_z\,. \tag{17}$$

Introducing the sub-amplitude $\mathcal{M}_{N-n}$ for $q_n + \bar{q}_B \to h_{n+1} + \cdots h_N$, we have

$$\mathcal{M} \equiv \mathcal{M}_N = \mathcal{M}_{N-n}\,T_n \cdots T_2\,T_1\;. \tag{18}$$

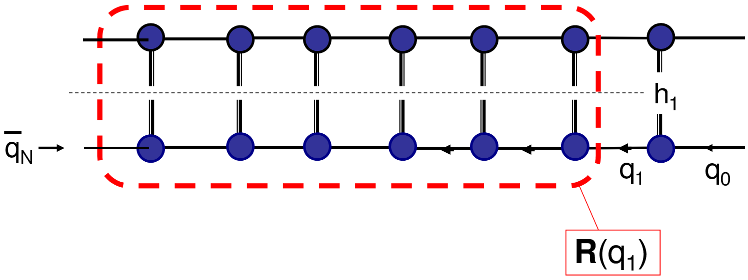


Figure 5: Unitarity diagram for $\mathcal{R}$.

- Using (18), the n-particle inclusive cross section with polarized quarks writes

$$\frac{d\sigma(q_A+\bar{q}_B\to h_1,\cdots h_n+X)}{d^3\mathbf{p}_1/p_1^0\cdots d^3\mathbf{p}_n/p_n^0}=\mathrm{Tr}\{\,\rho_0\,T_1^\dagger\,T_2^\dagger\cdots T_n^\dagger\,\mathcal{R}\{q_n\}\,T_n\cdots T_2\,T_1\}\,, \tag{19}$$

where ρ_0 is the spin density matrix of q_A and

$$\mathcal{R}\{q_n\}=\sum_{N>n}\int\frac{\mathrm{d}^3\mathbf{p}_{n+1}\cdots\mathrm{d}^3\mathbf{p}_N}{p_{n+1}^0\cdots p_N^0}\,\mathcal{M}_{N-n}^\dagger\,\sigma_z\,\frac{1-\sigma.\mathbf{S}(\bar{q}_B)}{2}\,\sigma_z\,\mathcal{M}_{N-n} \tag{20}$$

is the *cross section matrix* [11] of the reaction $q_n+\bar{q}_B\to$ hadrons. It operates in the spin space of q_n. It also depends on the antiquark polarization $\mathbf{S}(\bar{q}_B)$, but at large $(q_n+\bar{q}_B)^2$ this dependence is negligible and we may take $\mathbf{S}(\bar{q}_B)=0$. Fig.5 represents the unitarity diagram giving $\mathcal{R}\{q_A\}$. Encircled in dashed line is the unitarity diagram for $\mathcal{R}\{q_1\}$. The general cross section matrix $\mathcal{R}\{q\}$ satisfies the integral *recursion relation*

$$\mathcal{R}\{q\}=\sum_h\int\frac{\mathrm{d}^3\mathbf{p}}{p^0}\,T^\dagger\{q',h,q\}\,\mathcal{R}\{q'\}\,T\{q',h,q\}\,. \tag{21}$$

- We assume the following Regge behavior at large $(q+\bar{q}_B)^2$:

$$\mathcal{R}\{q\}\sim|(\bar{q}_B)^-q^+|^{\alpha_{\rm out}}\,\left[\beta_q(\mathbf{q}_{\rm T}^2)+\gamma_q(\mathbf{q}_{\rm T}^2)\,\sigma_z\sigma.\mathbf{q}_{\rm T}\right]. \tag{22}$$

$\alpha_{\rm out}$ is the *output Regge intercept.* $a_{\{q\}}$ of Eq.(4) and $\alpha_{\{q\}}$ of Eq.(10) are linked by [9]

$$a_{\{q\}}=\alpha_{\rm out}-2\,\Re\mathrm{e}\,\alpha_{\{q\}}\,. \tag{23}$$

A preliminary numerical task consists in calculating $\alpha_{\rm out}$ and the *Regge residue functions* $\beta_q(\mathbf{q}_{\rm T}^2)$ and $\gamma_q(\mathbf{q}_{\rm T}^2)$, solving the integral equation(21).

- Suppose that we know the flavor and momentum of quark $\{q_{n-1}\}\equiv\{q\}$ and its polarization $\mathbf{S}_{n-1}\equiv\mathbf{S}$. From Eqs.(21) and (22), one can write

$$\sigma\{q+\bar{q}_B\}=\mathrm{Tr}\,\{\rho\,\mathcal{R}\{q\}\}=|(\bar{q}_B)^-|^{\alpha_{\rm out}}\sum_h\int\frac{\mathrm{d}^3\mathbf{p}}{p^0}$$
$$(q'^+)^{\alpha_{\rm out}}\;\mathrm{Tr}\left\{\,T\{q',h,q\}\,\rho\,T^\dagger\{q',h,q\}\,[\beta_{q'}(\mathbf{q}_{\rm T}'^{\,2})+\gamma_{q'}(\mathbf{q}_{\rm T}'^{\,2})\,\sigma_z\sigma.\mathbf{q}'_{\rm T}]\,\right\} \tag{24}$$

with $p+q'=q$ and $\rho=(1+\sigma.\mathbf{S})/2$. The second line is proportional to the probability that quark $\{q\}\equiv\{q_{n-1}\}$ emits a hadron $\{h_n\}$ of species h and 4-momentum p. In the Monte-Carlo method, one generates h and p at random according to this probability. $\{q'\}\equiv\{q_n\}$ is related to $\{h\}$ by the conservation of charge, strangeness and 4-momentum.

- Once the flavors and momenta of $\{p\} \equiv \{p_n\}$ and $\{q'\} \equiv \{q_n\}$ are known, the q' polarization is given by

$$\frac{1+\sigma.\mathbf{S}'}{2} \equiv \rho' = \frac{T\{q',h,q\}\ \rho\ T^\dagger\{q',h,q\}}{\mathrm{Tr}\,(\ T\{q',h,q\}\ \rho\ T^\dagger\{q',h,q\}\ )}\ . \tag{25}$$

Thus one has obtained $\{q_n\}$ and $\mathbf{S}_n$. Iterating the last two steps, one generates the jet of a polarized quark.

6 Conclusion

We have given the principle of a recursive quark fragmentation model which includes the spin degree of freedom. Since spin has essentially quantum properties, we started from *amplitudes* rather *probabilities*. For that we took the amplitudes which underly the symmetric Lund fragmentation model.

When an imaginary part is given to the quark mass μ, the model produces the spin asymmetries of Collins and jet handedness, like in the toy model of [1] but with hadron mass shell contraints duly taken into account. For the moment we have no theoretical justification for taking a complex μ, but it provides a quantum realization of the string + $^3P_0$ mechanism, which up to now is in qualitative agreement with experiment.

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